Dynamic properties of photonic crystals and their effective refractive index

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The reflection and refraction properties of light at an interface between two media are usually classified as kinematic and dynamic. Both are determined by the refractive indices of the media. The kinematic properties refer to the direction of light propagation, whereas the dynamic properties refer to the polarization, magnitude, and phase changes of the reflected and refracted waves. Metamaterials and photonic crystals are often assigned an effective refractive index defined by their dispersion curves. This work shows for the first time, to our knowledge, that although the kinematic properties are consistent with this index definition, in some cases the dynamic properties are not. This observation has important implications for photonic crystal understanding and design because it shows that their rich physical phenomena cannot always be simplified to a description in terms of an effective refractive index. © 2005 Optical Society of America

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1. INTRODUCTION

The refractive index is a macroscopic material property that determines how light reflects and refracts at the interface between two media. Jackson classifies the phenomena of reflection and refraction into two classes: kinematic and dynamic. The kinematic properties describe how light propagation changes direction at an interface and are a direct consequence of the conservation of linear momentum. The dynamic properties describe the intensities, phase changes, and polarization of reflected and refracted waves through the Fresnel formulas and are a consequence of the continuity of the electromagnetic field (tangential components of the electric and magnetic fields).

Just as a crystal is a periodic array of atoms or molecules, a photonic crystal is a subwavelength periodic array of different materials. A photonic bandgap material is a photonic crystal in which light of certain frequencies cannot propagate in one or more directions. Because of their intriguing optical properties, photonic crystals are being studied for an increasing number of applications in terms of their optical properties from the nanometer-scale to microwave frequencies. They derive their properties from the subwavelength structure of their component materials in a similar way that homogeneous dielectrics derive their optical properties from the nanometer-scale structure of their atoms. When the wavelength of the field interacting with the structure is much longer than the unit cell, the metamaterial can be treated as a homogeneous dielectric with macroscopic parameters such as effective refractive index \( n_{\text{eff}} \). Proper choice of component materials and geometries can yield metamaterials with novel optical properties to control light in unconventional ways.

Traditional methods of deriving an effective refractive index apply in the long-wavelength limit. However, metamaterials consisting of thin wires can have optical properties of a low-loss dielectric at wavelengths much larger than the wire thickness, but only slightly longer than twice the unit cell. Recently, photonic crystals have also been assigned an effective refractive index based on their band structure at wavelengths only slightly longer than the unit cell.

The purpose of this paper is to analyze the limits of applicability of the different definitions of effective refractive index. We compare three methods for defining and computing the effective refractive index of photonic crystals when the long-wavelength limit does not apply, that is, when the free-space wavelength \( \lambda_0 \) is not much greater than the unit cell \( (\lambda_0 \lesssim a) \). The first method uses the angle-dependent reflectivity \( R(\theta) \) at the interface of a finite structure, leading to \( n_{\text{eff}}^d \). The second considers the normal-incidence refraction into a finite structure, leading to \( n_{\text{eff}}^r \). Finally, the third method derives the index from the dispersion curve of an infinite periodic structure, leading to \( n_{\text{eff}}^d = c_0 k / \omega \). In some instances, the index \( n_{\text{eff}}^d \) is constant regardless of the propagation direction, so it defines a circular equifrequency surface (EFS) in \( k \) space for two-dimensional photonic crystals.

The new results presented here are summarized as follows: In some cases, such as in silver nanowire arrays at visible wavelengths, \( n_{\text{eff}}^d \) predicts both its kinematic and its dynamic properties: \( n_{\text{eff}}^d = n_{\text{eff}}^R = n_{\text{eff}}^d \); however, for certain dielectric photonic crystals operating near the band edge, \( n_{\text{eff}}^d \) predicts only their kinematic properties. Specifically,
as a case study, we consider several photonic crystals for which \( n_{\text{eff}}^2 < 1 \), as they exhibit several interesting properties.

This paper is organized as follows. In Section 2 we review methods of defining \( n_{\text{eff}} \) both in the long-wavelength limit and close to resonance, when \( \lambda_0 \approx a \). In Section 3 we use the three methods mentioned above to find the effective refractive index of several photonic crystal structures for which \( n_{\text{eff}} < 1 \). Finally, in Section 4 we discuss the limitations of the different definitions and their physical explanation. In Section 5 we present the conclusions of this analysis.

2. DEFINING EFFECTIVE INDEX

A. Classical Theory and the Long-Wavelength Limit

To begin a discussion of how to determine the effective refractive index of a periodic structure with subwavelength feature sizes, it is appropriate to review the standard derivation of the refractive index of a homogeneous isotropic dielectric, which results in the Clausius–Mossotti equation. Also known as the Lorentz–Lorenz relation, it expresses the dielectric constant \( \varepsilon \) of a homogeneous material as a function of the molecular density \( N \) and polarizability \( \alpha \): \((e-1/e+2)=(4\pi/3)N\alpha\). This formula works best for dilute gases and is a good approximation for isotropic dielectrics with low dielectric constants.13,14

Now consider a subwavelength periodic structure with \( a < \lambda_0 \) consisting of \( N \) materials, denoted by subscript \( i \), with dielectric constant \( \varepsilon_i \), density \( N_i \), and volume fraction \( f_i \) where \( \sum f_i = 1 \). Finding the effective dielectric constant involves simply replacing \( N \alpha \) in the Clausius–Mossotti equation with \( \sum f_i \varepsilon_i N_i \alpha_i \) and then expressing this in terms of the component dielectric constants and fill factors. This results in the Lorentz–Lorenz effective-medium approximation, which was generalized by Maxwell–Garnett and then Bruggeman. The most general expression is15,16

\[
\frac{n_{\text{eff}}^2 - 1}{n_{\text{eff}}^2 + 2} = \sum_{i=1}^{N} f_i \frac{1}{\varepsilon_i + 1}.
\]

The above derivation does not account for the specific geometry of the structure. Given the dielectric constants and fill factors of the component materials, their orientation with respect to the electric field is the next important factor determining the structure’s effective dielectric constant. Components of the electric field normal to surfaces induce a polarization charge density on them, which then screens the surface’s effect on the field from other parts of the medium and reduces the influence of that feature on the effective dielectric constant.17

In one limiting case, the structure is a multilayer stack where all interfaces are parallel or perpendicular to the applied field. To first order, this effective-medium approach yields the same effective dielectric constants as those derived using the field continuity for structures exhibiting form birefringence: \( \varepsilon_i = \varepsilon_i \langle f_i \rangle + \varepsilon_0 \langle f_i \rangle / \varepsilon_0 \). These expressions, known as the Wiener bounds, define the range of effective dielectric constant values for a two-component subwavelength structure of any geometry. Given the fill factors, the range of effective index values narrows to what is known as the Hashin–Shtrickman bounds. Other geometric characteristics further narrow the range of accessible \( n_{\text{eff}} \) values.13,17,18

Artificial dielectrics have been typically composed of arrays of inclusions such as conducting wires, strips, spheres, disks, or plates designed for lightweight microwave lenses. The above classical methods have been used to derive the effective properties of such structures, but transmission-line analysis became popular because of its applicability and familiar mathematical form gave additional insight into the physics of the problem.19,20 Typically, the operating wavelength for artificial dielectrics is much larger than both the unit cell \( a \) and the inclusion width. Several other methods for the analysis of artificial dielectrics have since become popular. These include analytical methods known as homogenization techniques and effective-medium theory. They have been developed to find the effective optical properties of layered media,16 subwavelength gratings,21,22 and dielectric photonic crystals.23,24 Homogenization of metal photonic crystals with thin wires25,26 has resulted in analytical expressions of the dielectric constant resembling those derived with transmission-line techniques.20 Numerical methods of computing effective optical properties include Fourier expansion methods.27,28

The above methods of defining effective refractive index apply either in the long-wavelength limit, where \( a \ll \lambda_0 \), or when the largest inclusion (with dimension \( b \)) in a host medium is much smaller than the unit cell (\( \beta \ll a \)). The latter condition allows the wavelength to be just slightly larger than the unit cell. Yet several recent studies have reported photonic crystals5,8–11,29–31 that exhibit properties of homogeneous dielectrics but satisfy only one or neither of these conditions. Subsections 2.B–2.D discuss methods of defining the effective refractive index in these cases.

B. Equifrequency Surfaces of Periodic Structures

Another method of defining an effective refractive index is by analyzing EFSs, the wave-vector surface defined by modes of a single frequency.9–11

For light propagating in the \( x-y \) plane, a homogeneous dielectric material has a wave-vector surface that satisfies the relation \( k_x^2 + k_y^2 = n_x k_0^2 \) where \( k_0 = \omega / \varepsilon_0 \). If the wave-vector surface of a two-dimensional photonic crystal has a circular EFS satisfying \( k_x^2 + k_y^2 = n_{\text{eff}}^2 k_0^2 \), then it has an effective refractive index \( n_{\text{eff}}^2 \) at frequency \( \omega = \varepsilon_0 k_0 \). This is a phase refractive index and the material is considered isotropic at that frequency. To define an effective index for light propagating along the symmetry axis of a periodic structure, one needs to consider only the photonic band structure, which is simply a subset of the \( \omega(k) \) values used to generate EFSs such that the wave vector \( k \) is on a symmetry axis. In this paper we will distinguish between \( n_{\text{eff}}^d \), which derives from a circular EFS and holds for all directions, and the band index \( n_{\text{eff}}^d \), which applies to a single propagation direction.
C. Finite Structures: Reflection and Refraction

So far we have discussed methods of defining effective refractive index of an infinite subwavelength periodic structure. Yet, for real applications, the refractive index of a material also manifests itself at its boundary with another medium, that is, it determines reflection, transmission, and refraction of light at an interface. Therefore there are two other methods to define the effective refractive index, according to reflection and refraction.

A definition of effective index consistent with the dynamic properties at interfaces between homogeneous materials and photonic crystals should provide the proper reflection and transmission coefficients as predicted by Fresnel formulas.

A single-angle method defines the complex refractive index of a photonic crystal as a function of its normal-incidence reflection and transmission coefficients in such a way that it provides the same properties as a homogeneous structure with the corresponding effective refractive index.

Alternatively, in the case of isotropic photonic crystals, the Fresnel formulas can be used to find the complex refractive index of a homogeneous dielectric whose angle-dependent reflectivity R(θ) best matches that of the photonic crystal. Compared with the single-angle method, this multiple-angle method is more robust because the computed n_eff is calculated from the reflectivity at various angles.

Moreover, a photonic crystal with an effective refractive index should also refract light like a homogeneous structure with the corresponding refractive index. If the structure’s features are small enough compared to the unit cell and wavelength, an incident plane wave will be refracted as a mode that is very similar to a plane wave, and the effective index will be n_eff = λ_0/λ, where λ is the wavelength inside the photonic crystal. To the extent that the propagating Bloch wave in the photonic crystal resembles a plane wave, calculating the wavelength inside the photonic crystal provides an additional method for estimating the effective index.

D. Phase and Group Indices

Up to now we have discussed the effective phase refractive index of photonic crystals and not the group refractive index. The group refractive index is defined as n_g = v_g/c_0, where the group velocity is v_g = dv/dk. Therefore one method of finding n_g is by computing the slope of the dispersion curve, which provides the group velocity.

Another method to calculate the group effective index is by calculating the transmission through a Fabry–Perot resonator composed of the photonic crystal under study. The peak transmission frequency spacing of a Fabry–Perot resonator is related to its group refractive index n_g. Previous research has shown that the group index calculated by this method agrees with the group index calculated from the band diagram.

3. CASE STUDY: PHOTONIC CRYSTALS WITH A BAND INDEX OF LESS THAN ONE

A. Silver–Air Thin Wire Metamaterial

We start by applying the various definitions of phase effective index to a metamaterial composed of silver wires embedded in air. Because of the losses of silver at optical frequencies, these metamaterials are inherently lossy and present a complex refractive index. Reference 5 showed that metal dielectric metamaterials can be tailored to present the real part of the effective index below unity with the imaginary part significantly lower than the bulk metal.

Here we analyze a two-dimensional metamaterial with a square array of cylindrical silver wires embedded in air. The unit cell is a = 200 nm and the wire radius is r = 15 nm. Therefore this metamaterial does not fall in the long-wavelength regime at optical frequencies. However, it behaves on reflection, refraction, and transmission as a low-loss dielectric with 0 < Re(n_eff) < 1 for wavelengths between 0.45 and 1.2 μm for light polarized parallel to the wires, as shown in Fig. 1.

We calculated the effective refractive index in three ways: first by using the band diagram (n_eff), second by using reflection data matched at all angles of incidence (n_eff), and third calculating the wavelength inside the metamaterial (n_eff). The computations of n_eff and n_eff were performed using a finite-element method and n_eff was calculated using a transfer-matrix method.

As shown in Fig. 1, the three independent calculations provide consistent results for both the real and the imagi-
nary parts of the index in a wide frequency range for the two main directions \( \Gamma - X \) and \( \Gamma - M \). The band diagram (Fig. 2) shows the real part of the eigenfrequencies corresponding to \( 0 < \Re(n_{\text{eff}}) < 1 \). Each eigenfrequency was computed iteratively considering the frequency-dependent refractive index of silver, \( n(\omega) \).

Figure 2 shows that, for frequencies corresponding to free-space wavelengths \( \lambda_0 = \frac{2a}{\pi} \), the mode is above the light line, which suggests that the wire array has an effective refractive index \( n_{\text{eff}} = \frac{c\kappa}{\omega} \) less than unity. This effective refractive index agrees with those predicted for reflection and refraction of finite structures, as shown in Fig. 1. Unlike lossless periodic media, where the imaginary part of the index, obtained from eigenfrequency calculations, is related to backreflection, here it is also related to attenuation.

Figure 3 shows the angle-dependent reflectivity of a slab of metamaterial and compares it with that of a hypothetical homogeneous material with the same index of refraction. The apparent agreement justifies use of the angle-dependent method for the definition of an effective index \( n_{\text{eff}}^R \).

**B. Hexagonal Photonic Crystals**

We now focus on dielectric photonic crystals that can also be designed to present a phase effective index below unit based on the equifrequency diagram. In this subsection we use two designs proposed by Notomi in Ref. 9. These designs have unit cells comparable to the wavelength, and therefore the long-wavelength approximations do not apply. The band diagrams and effective refractive-index values derived from them were calculated with a freely available software package. Although there is a slight disagreement with the band diagrams reported in Ref. 9, it is irrelevant to our purposes.

The first photonic crystal is an hexagonal array of cylinders \( n_{\text{cyl}} = 3.6 \) with a diameter of \( 2r = 0.7a \), where \( a \) is the unit cell length. For TE modes (electric field normal to the cylinders), this structure has a circular EFS for \( a/\lambda_0 \) between 0.59 and 0.62. These frequencies correspond to band I and band II in Fig. 4(a). For band I, \( 0 < n_{\text{EFS}}^{\text{I}} < 0.5 \); for band II, \( -0.8 < n_{\text{EFS}}^{\text{II}} < 0 \), as shown in Fig. 4(b).
Reference 9 indicates that the effective index defined by a circular EFS is consistent with Snell’s law. Here we investigate whether this index is consistent with the dynamic properties, i.e., the Fresnel formulas. For this purpose we calculated the reflectivity from air at \( a/\lambda = 0.62 \) as a function of the angle of incidence on a 64-period wide photonic crystal slab using the transfer-matrix method.\(^{37} \)

Figure 5 shows the result and a comparison with the reflectivity of a hypothetical homogeneous material with the same index. Clearly, the photonic crystal does not reflect like a \( n < 1 \) dielectric.

This behavior is the consequence of the interplay between the photonic band structure, which applies for an infinite crystal, and the mode-matching problem, which applies to the interface between the two media. In effect, although some modes can be coupled from plane waves, some cannot. In contrast, metamaterials in the long-wavelength regime couple plane waves into modes that are basically plane waves and thus obey the same rules as homogeneous materials.

To illustrate this issue, we compare the band diagram and the transmission spectra. The symmetry of the modal field distributions explains in part the transmission properties of each mode. The modes are either symmetric or antisymmetric about the axis connecting the cylinder centers corresponding to the direction of propagation. For \( \Gamma - M \) modes of a hexagonal lattice of unit cell \( a \), the axis connects the second-nearest neighboring cylinders, whereas for the \( \Gamma - K \) modes, the axis connects adjacent cylinders. As mentioned above, an incident plane wave propagating along the symmetry axis can couple only into symmetric modes. As shown in Figs. 6 and 7, the zero-transmission regions correspond to antisymmetric modes.

The second example of photonic crystal with circular EFS is composed of a hexagonal array of air holes in a host medium \((n_{\text{host}} = 3.6, 2r = 0.8a)\).\(^9\) For TM modes (electric field parallel to the cylinders), this structure has a circular EFS for frequencies in band I and band II in Fig. 8(a), corresponding to an effective index \(-1 < n_{\text{EFS}}^{\text{eff}} < 0\) for band I and \(0 < n_{\text{EFS}}^{\text{eff}} < 0.5\) for band II, as shown in Fig. 8(b).

Again, the angle-dependent reflectivity (Fig. 9) shows that, even for the symmetric modes, the photonic crystal does not have the reflectivity of a homogeneous dielectric with \( n < 1 \). Moreover, observing the band diagram (Fig. 8) and transmission spectra (Figs. 10 and 11) we find zero transmission for band I and II frequencies incident in the \( \Gamma - K \) direction.

C. Square Photonic Crystals

Gralak et al.,\(^{10}\) presented a similar design of a photonic crystal with an effective refractive index composed of a square array of dielectric cylinders \((n_{\text{cyl}} = 3.0)\) with a diameter of \(2r = 0.748a\) where \(a\) is the unit cell length. For TM modes (electric field parallel to the cylinders), this structure has a circular EFS in a small frequency range close to the normalized frequency \(\Omega_1 = (v_p a)/c_0 = 0.496\) just above the bandgap.

Figure 12(a) shows the band diagram of this photonic crystal calculated using finite-element software.\(^{36}\) The circular EFS at \(\Omega_1\) corresponds to the fourth band, as noted in the band diagram. As shown in the inset, both of
these modes, unlike the hexagonal photonic crystal discussed above, are symmetric at $\Omega_1$ and hence can be excited by an incident plane wave. The second intersection of the fourth mode at $\Omega_1$ in the $\Gamma\rightarrow M$ section, near the $\Gamma$ point, is antisymmetric and hence cannot be excited by an incident plane wave.

The band index $n_{\text{eff}}^d$ plotted in Fig. 12(b), shows the same band index for $\Gamma\rightarrow X$ and $\Gamma\rightarrow M$ propagation directions at frequencies just above the bandgap. Note again that these index values differ slightly from those reported in Ref. 10 and are calculated with a different numerical method.

Since the photonic crystal modes at $\Omega_1$ are both symmetric, one could expect that it would exhibit angle-dependent reflectivity consistent with the Fresnel formulas, including total external reflection (TER). However, the critical angle of a dielectric with refractive index $n < 0.1$ is less than 5.7°. Yet, as shown in Fig. 13, the reflectivities at $\Omega_1$ are less than one at several angles exceeding the critical angle for both $\Gamma\rightarrow X$ and $\Gamma\rightarrow M$ orientations.
Moreover, TER does not even occur at neighboring frequencies, as the transmission of a finite slab for light incident at 45° is nonzero and independent of the slab thickness, as shown in Fig. 14. Since changing the slab length shifts frequencies of each transmission peak, they are due to interference, not TER. As noted in Subsection 2.D, the transmission frequency spacing corresponds to the group refractive index.

Although this photonic crystal does not have an effective index consistent with its dynamic reflection properties, Gralak et al.\textsuperscript{10} show that it obeys Snell’s law, and hence the kinematic properties are consistent with a dielectric with \( n < 1 \).

4. DISCUSSION

Quantitatively, refraction and reflection from homogeneous materials are described by kinematic properties (like Snell’s law) and dynamic properties (Fresnel formulas). The refractive index plays a critical part in both cases. In the case studies presented above we have analyzed these properties in photonic crystals at (free-space) wavelengths comparable to the unit cell size. For the silver nanowire photonic crystal, the band index predicts both its kinematic and its dynamic optical properties. However, for the dielectric photonic crystal, the band index, even if associated with circular EFSSs, predicts only their kinematic properties provided that plane waves can couple into the mode for which the band index is defined.

The effective index discussed for the dielectric photonic crystals belongs to the short-wavelength regime (\( \lambda_0 \approx a \)). The fields inside the photonic crystals are Bloch waves with a high degree of complexity because they belong to higher bands. They are thus different from plane waves. Because the Fresnel formulas are derived from the plane-wave solutions to Maxwell’s equations, we should not ex-

\[ \text{Fig. 12. (a) Dispersion diagram (TM modes, electric field is parallel to cylinder axis) of a two-dimensional square photonic crystal, unit cell } a, \text{ consisting of dielectric cylinders of radius } r=0.374a \text{ and refractive index } n=3.0. \text{ The insets show the electric field of modes with frequency } a/\lambda_0=0.496. \text{ (b) Effective refractive index } n_{\text{eff}} \text{ above the bandgap.} \]

\[ \text{Fig. 13. Reflectivity of finite-thickness slabs in vacuum. The electric field is parallel to the cylinder axes and normal to the plane of incidence (s polarization). Top: A 64-period photonic crystal slab consisting of a square array of cylinders (n=3) at normalized frequency } a/\lambda_0=0.4956. \text{ Bottom: A homogeneous dielectric slab with refractive index } n=0.04 \text{ and the same thickness as the photonic crystal slab.} \]

\[ \text{Fig. 14. Shown us a 45° incidence transmission through 512 and 513 periods of square photonic crystal described in Fig. 12, } \Gamma-\text{X direction. Since the transmission minima depend on the length of the photonic crystal slab, none of them are due to TER occurring at a specific frequency.} \]
pect them to apply to structures that support Bloch waves, and they cannot be used for impedance matching to photonic crystals unless proven otherwise. Mode symmetry, for example, plays an important role at interfaces between homogeneous materials and photonic crystals. Modes are either symmetric or antisymmetric about the photonic crystal symmetry axis. Consider a plane wave incident on a finite photonic crystal such that $k$ is parallel to the symmetry axis of the photonic crystal. Since the incident wave is symmetric about this axis, it will excite only a symmetric mode. If the only mode at this frequency is antisymmetric, all light will be reflected, which would contradict the Fresnel equations if a nonzero $n_{\text{eff}}$ was used.

Nevertheless, in the silver wire photonic crystal, with wires much smaller than both the unit cell and the free-space wavelength, the propagating Bloch modes strongly resemble plane waves (Fig. 2). In this case the Fresnel formulas are a good approximation to predict the metamaterial's dynamic optical properties. Since this effective index predicts both its kinematic and its dynamic optical properties, it can be used in an effective-medium approach, even though the metamaterial is outside of the long-wavelength limit. This suggests that in the limit of small inclusions, such as thin cylinders, in a host material with the unit cell only slightly smaller than half of the free-space wavelength, the modes corresponding to the lowest bands can be excited and associated with an effective index that is consistent with the dynamic properties. This behavior can be observed regardless of the existence of a bandgap at higher frequencies.

We have analyzed the dynamic properties of photonic crystals at the interface with a homogeneous material and observed that they cannot always be described in terms of an effective (complex) refractive index. The main implication is that, in general, the nature of wave phenomena generated by photonic crystals cannot be reduced to a description in terms of a single number, i.e., the effective index.

Even if the kinematic properties of a photonic crystal can be predicted by an effective index derived from the band diagram, the effectiveness of any application of the photonic crystal will depend on its dynamic behavior. For example, if a device is designed for a specific purpose but no modes can be excited, its utility would be hampered. This should not be understood as a drawback of photonic crystals but rather as a shortcoming of an oversimplified model. In fact, a photonic crystal design could outperform in terms of transmissivity a homogeneous material design having the same band index.

On the other hand, in the cases for which an effective index can describe both the kinematic and the dynamic properties, the photonic crystal can be used as a building block for more complicated structures and still be modeled (at least in first approximation) as a homogeneous material.

5. CONCLUSIONS

We investigated the different definitions of effective refractive index and their limits of applicability. As an example we showed that the reflection and refraction at the interface between air and certain metamaterials are consistent with a unique refractive index. In contrast, the dynamic properties of certain photonic crystals with frequencies near the band edge cannot be simplified to a description in terms of a unique effective refractive index, even though their kinematic properties can.

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REFERENCES

36. FEMLAB, Comsol Corporation, Burlington, Mass.
37. A. L. Reynolds, Translight Software (Optoelectronics Research Group, University of Glasgow, 2000).